Code No: 111AB **R13**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year Examinations, January/February - 2024

MATHEMATICS - I

(Common to CE, EEE, ME, ECE, CSE, IT, MCT, AE, MIE, AGE)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART - A

(25 Marks)

1.a) Define the rank of a matrix. [2]

b) Consider the system of linear equations:

$$\begin{pmatrix} 1 & a & 1 \\ 1 & 2 & 2 \\ 1 & 5 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ b \\ 9 \end{pmatrix}$$

For what values of a and b, the system has:
(i) unique solution, (ii) infinite solutions, and (iii) no solution. [3]

c) If $x = r\cos\theta,\ y = r\ \sin\theta$, find $J\left(\frac{x, y}{r, \theta}\right)$. [2]

d) State Rolle's Mean value theorem and give an example of function for which Rolle's theorem can't be applied. [3]

e) Apply Beta or Gamma Function to evaluate: $\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\, d\theta$. [2]

f) Apply Beta or Gamma Function to evaluate: $\int_0^{\frac{\pi}{2}} \tan^n x\, dx$ for $-1 < n < 1$. [3]

g) Explain Law of natural growth/decay. [2]

h) The general form of a differential equation is given by:

$\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1}y}{dx^{n-1}} + a_2 \frac{d^{n-2}y}{dx^{n-2}} + \cdots + a_n y = V$

Where $a_1, a_2, \cdots, a_n$ are constants and V is a function of x only. Then, write down the expression for the general solution of this differential equation when all the roots of the auxiliary equation are real and different. [3]

i) Determine $\mathcal{L}\left\{\frac{2\sinh 2t}{2}\right\}$. [2]

j) Determine $\mathcal{L}\left\{e^{-2t}\int_0^t e^{2\tau}\cos(3\tau)d\tau\right\}$. [3]

PART - B

(50 Marks)

2. Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & 2 & 1 \\ 2 & 1 & -3 & 0 \\ 1 & 1 & 2 & 1 \end{bmatrix}$. [10]

(Don't use decimal numbers to find the inverse.)

OR

3.a) Given $A = \begin{pmatrix} 1 & 1 & -1 & 3 \\ 2 & -2 & 6 & 8 \\ 3 & 5 & -7 & 8 \end{pmatrix}$. Then, find the rank (AA^T).

b) Use Gauss Elimination process to solve the system: [5+5]

$$\begin{pmatrix} -4 & 1 & 0 & 1 \\ 1 & -4 & 1 & 0 \\ 0 & 1 & -4 & 1 \\ 1 & 0 & 1 & -4 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} -200 \\ -300 \\ -300 \\ -200 \end{pmatrix}$$

4. Apply Lagrange's Multipliers method to find the shortest and longest distance between the line $y = 10 - 2x$ and the ellipse $(x^2/4) + (y^2/9) = 1$. [10]

OR

5. If $u = x + y^2,\ v = y + z^2,\ w = z + x^2$, then apply Jacobian concept to check the functional dependency of u, v, w. [10]

6. Apply triple integral to find the volume enclosed by the solid: [10]

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} + \left(\frac{z}{c}\right)^{2/3} = 1$$

OR

7. Find the area of that part of the surface of the paraboloid $y^2 + z^2 = 2ax$, which lies between the cylinder, $y^2 = ax$ and the plane $x = a$. [10]

8. Solve $(D^3 + 1)y = \cos^2\left(\frac{x}{2}\right) + e^{-x}$. [10]

OR

9. Solve $(D^2 + 5D + 6)y = e^{-2x} \sec^2 x\,(1 + 2\tan x)$. [10]

10. Determine the solution of the linear differential equations $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} - 3x = \sin t$, given that x and $\frac{dx}{dt}$ vanish at t = 0. [10]

OR

11. Determine $\mathcal{L}^{-1}\{F(s)\}$, where $F(s) = \frac{s^3 - 5s^2 + 6s + 7}{s(s-1)(s^3 + 6s^2 + 11s + 6)}$. [10]

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